Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #18

Integrals and Area - 1

Consider the integral:

$$\int_a^b f(x)\,dx$$

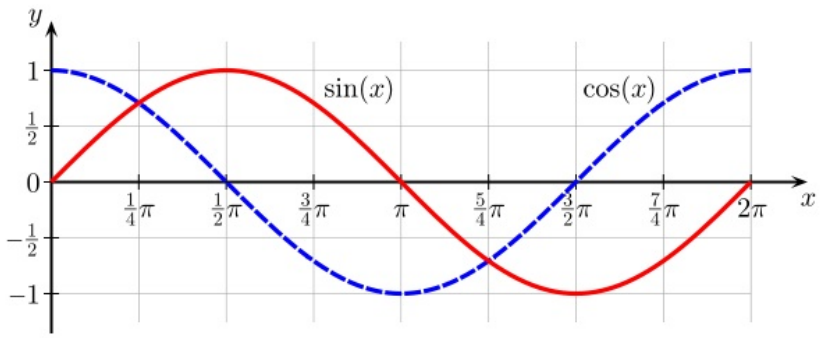
We can interpret this integral as an area.

Simple version:

$\int_a^b f(x)\,dx$ = the area under the graph of $y = f(x)$
from $x = a$ to $x = b$

Example:

$$\int_0^{\pi} \sin x\,dx$$



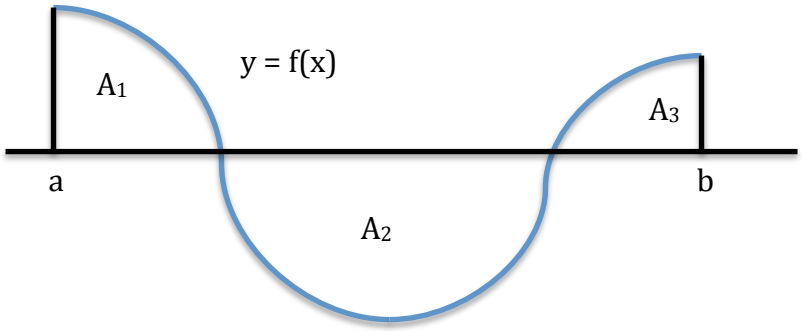
$$\int_0^{\pi} \sin x\ dx = (-\cos x)]_0^{\pi} = -\cos\pi + \cos 0 = 2$$

$$\int_0^{2\pi} \sin x\ dx = (-\cos x)]_0^{2\pi} = -\cos 2\pi + \cos 0 = 0$$

Better version:

$$\int_a^b f(x)\,dx$$

is equal to the sum of the area above the x-axis and below the curve $y = f(x)$ minus the area below the x-axis and above the curve $y = f(x)$.

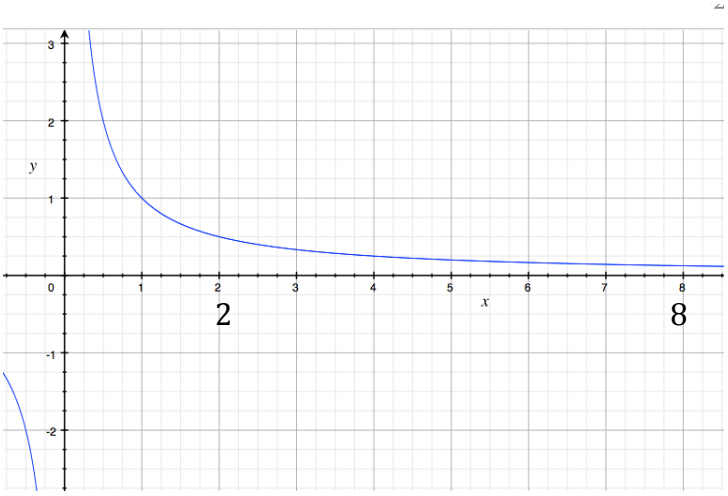


$$\int_a^b f(x)\ dx = A_1 - A_2 + A_3$$

Example:

Calculate the area under the graph of $y = \frac{1}{x}$ between $x = 2$ and $x = 8$.

Step 1: Sketch the graph.



Step 2: Calculate

Area =

$$\int_2^8 \frac{1}{x} dx = \log x]_2^8 = \log 8 - \log 2 = 1.39$$